LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2018**

16/17/18PST1MC03/ST 1822 – STATISTICAL MATHEMATICS

Date: 30-10-2018 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

SECTION – A

Answer ALL questions. Each carries TWO marks. (10 x 2 = 20 marks)

1. Let s = 1, -1, 1, -1, ... and let σ(i) = 2i – 1 for all i ϵ N. Check whether s $\circ$ σ is a subsequence of s. Further if $\sigma(i) = 4^i$, write down the subsequence.
2. Prove that the sequence (n) where n ϵ N does not have a limit.
3. If (s_n) converges to L $\neq$ 0, prove that ($(-1)^n s_n$) oscillates.
4. State the Limit form of the comparison test for the series of positive terms.
5. Define absolute convergence and conditional convergence of a series of real numbers.
6. Find sup f(x) and inf f(x) for the function $f(x) = e^{-|x|}$ on (-∞, ∞).
7. Prove that $\lim_{x \to 2}(2x - 1) = 3$.
8. Let f(x) = |x| for x ∈ (-∞, ∞). Show that f does not have a derivative at 0, even though f is continuous at 0.
9. State First Fundamental Theorem of Calculus.
10. Define Basis and Dimension of a vector space.

SECTION - B

Answer any FIVE questions. Each carries EIGHT marks. (5 x 8 = 40 marks)

11. exists, then show that it is unique.
12. Prove that the sequence (s_n) where s_n = 0 when n is odd and s_n = 1 when n is even does not converge.
13. State and prove Cauchy Criterion of Convergence of a series.
14. Check for the convergence of the series $\sum_{n=0}^{\infty} x^n$ if (i) 0 < x < 1, and (ii) x ≥ 1.
15. Prove that the series $\sum(-1)^n$ $[\sqrt{n^2 + 1}$ - n] is conditionally convergent.
16. If f and g are both bounded on A and c is any real number, then show that the functions f + g, cf, and f.g are each bounded on A.
17. If f ∈ R[a, b], then prove that |f| ∈ R[a, b] and $|\int_a^b f| \leq \int_a^b |f|$.
18. State and prove the First Mean Value Theorem of Integral calculus.

SECTION – C

Answer any TWO questions. Each carries TWENTY marks. (2 x 20 = 40 marks)

19(a) Find $\lim_{n \to \infty} c^{1/n}$ where c is a fixed positive number. (10)

19(b) By Leibnitz test, verify the convergence of the series:

(i) $\sum \frac{(-1)^{n+1}}{\log(n+1)}$ (ii) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ (10)

20. Let $f(x) = \frac{1}{x}$. Then show that (i) f is unbounded for $0 < x < \infty$ (ii) $\inf_{x > 0} f(x) = 0$

(iii) f is bounded on (a, ∞) for any $a > 0$. (20)

21(a) State Comparison Test for convergence of the improper integrals of the first kind. Hence verify the convergence of $\int_a^\infty \frac{1}{e^x + 1} dx$. (10)

21(b) Describe μ - Test for Convergence of integral of first kind and test for convergence of

(i) $\int_0^\infty \frac{x^2 dx}{(k^2 + x^2)^2}$ (ii) $\int_0^\infty \frac{x^3 dx}{(k^2 + x^2)^2}$ (10)

22(a) State Taylor's formula and Maclaurin's Theorem with Lagrange's Form of Remainder. Hence write down Taylor's formula for f(x) = log(1 + x) about a = 2 and n = 4. (10)

22(b) Explain the characteristic value problem and define the characteristic roots and vectors. Hence for the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, obtain the characteristic roots and vectors. (10)

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